Analytical and finite-element modelling of the incudostapedial joint

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Introduction

The joints of the ossicular chain significantly affect sound transmission through the middle ear, but the mechanical behaviour of these joints is not well understood. We previously analyzed the incudostapedial joint using a finite-element (FE) model with a simplified geometrical description of the pedicle of the lenticular process, based on histological serial sections and X-ray micro-CT, and using a priori estimates for material-property parameters, but we neglected the presence of synovial fluid (SF) in the joint (Funnell et al., MEMRO 2015). We later included SF in a FE model and made preliminary comparisons with experimental tension and compression measurements (Decraemer et al., MEMRO 2015).

Incudostapedial joint (ISJ) - a synovial joint

Experimental measurements on human incudostapedial joints under tension or compression (Zhang & Gan, 2011, Figs 4 & 6)

Note the hysteresis (not modelled here) and strong asymmetry
Our FE simulations done using FEBio (presented previously at MEMRO 2015, Aalborg) compared with Zhang & Gan model

Analytical model with greatly simplified geometry

Synovial fluid and cartilage on incus and stapes bone modeled as thin cylindrical slab with circular cross section
ISJ capsule approximated as thin cylindrical tube surrounding the slab
we use the theory of large deformations of elastic membranes to analytically model ISJ
we use the theory of large deformations of elastic membranes to model the capsule
Core filled with incompressible (fluid-like) Mooney-Rivlin material with strain energy function

\[ W = C_1(I_1 - 3) + C_2(I_2 - 3) \]

Two cases studied: Fixed bony plates with varying volume and constant volume with varying pressure
Parameters: \( A_1 \): local stretch ratio in \( z \) direction at \( z = 0 \), \( A_2 = r/r_0 \): local stretch ratio in \( r \) direction (inflation) at \( z = 0 \); \( P \): internal pressure; \( C_1 \) & \( C_2 \): Mooney-Rivlin coefficients; \( L \): initial membrane length.

Results

Comparing analytical solutions (MATLAB) and FE solutions (FEBio)

Note that these FE models are different from the previous incudostapedial joint model. These are cylindrical membranes with the same geometry as the analytical model.

Tube profiles from analytical solution (solid lines) compared with FE calculations (symbols) at various inflations when bony plates are fixed without initial elongation (left) and with initial elongation (right).

Stability analysis

Top: Typical pressure-vs-inflation behaviour where equilibrium configurations at various pressure values are intersections of constant-pressure lines with pressure equilibrium path. Bottom: Energy of the same constant-pressure configurations showing whether the equilibriums are minima or maxima of energy. The maximum pressure limit corresponds to inflection points in the energy curves, meaning that the equilibriums are stable only before reaching the maximum pressure.

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