APPLICATION OF THE TAGUCHI METHOD TO SENSITIVITY ANALYSIS OF A MIDDLE-EAR FINITE-ELEMENT MODEL

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ABSTRACT

Sensitivity analysis of a model is the investigation of how outputs vary with changes of input parameters, in order to identify the relative importance of parameters and to help in optimization of the model. The one-factor-at-a-time (OFAT) method has been widely used for sensitivity analysis of middle-ear models. The results of OFAT, however, are unreliable if there are significant interactions among parameters. This paper incorporates the Taguchi method into the sensitivity analysis of a middle-ear finite-element model. Two outputs, tympanic-membrane volume displacement and stapes footplate displacement, are measured. Nine input parameters and four possible interactions are investigated for two model outputs. For the tympanic-membrane volume displacement, the contributions from the Young's modulus and thickness of the pars tensa are dominant. For the footplate displacement, several input parameters play important roles and a strong interaction exists between the Young's moduli of the incudomallear and incudostapedial joints. It is important to take this interaction into account when the parameters' effects on model behaviour are being considered.

KEYWORDS

Sensitivity analysis; Taguchi method; Middle-ear finite-element modelling; Interaction

INTRODUCTION

Sensitivity analysis is an important step in building a model. As Fürbringer (1994) mused: "Sensitivity analysis for modelers? Would you go to an orthopedist who doesn't use X-ray?" Sensitivity analysis is the investigation of how outputs vary with changes of input parameters. This analysis can not only identify the relative importance of parameters, but also help in optimization of the model and in better understanding of the mechanisms of the studied system. The one-factor-at-a-time (OFAT) method has been widely used in middle-ear sensitivity analysis: each parameter is evaluated across a range of values while keeping other parameters constant. After all tests are performed, graphs are constructed to show how the output is affected by individual parameters. The difference in the model output due to the change in the input variable is referred to as the sensitivity. The relative importance of parameters is judged based on the magnitude of the calculated sensitivity. The OFAT method does not, however, take into account the possibility of interactions among parameters. Such interactions mean that the model sensitivity to one parameter can change depending on the values of other parameters.

Alternatively, the full-factorial method permits the analysis of parameter interactions, but generally requires a very large number of simulations. This can be impractical when individual simulations are time-consuming. A more practical approach is the Taguchi method, which is commonly used in industry. It employs only a small number of all the possible combinations of model parameters to estimate the main effects and some interactions. An orthogonal array (OA) is used to reduce the number of simulations (Taguchi, 1987) but still obtain reasonable information.

MATERIAL AND METHODS

A Middle-Ear Finite-Element Model

A 3-D finite-element model of an adult human middle ear was generated based on x-ray microscopic computed tomography (µCT) data with 19-µm voxels. The model's material properties were obtained from the literature (Funnell, 1996; Kirikae, 1960; Koike et al., 2002; Siah, 2002).In this study, only static simulations were conducted so inertial and damping parameters can be neglected. A static pressure of 1 Pa was applied to the surface of the tympanic membrane (TM). The finite-element simulations were done with SAP IV (Bathe et al., 1974). A detailed description of this model can be found elsewhere (Mikhael et al., 2004).

Taguchi Method

The procedure for applying the Taguchi method is as follows. Minitab™ 14 was used for the calculations.

Step 1: Select parameters and interactions of interest.
Step 2: Select parameter levels.
Step 3: Find a suitable OA with the smallest number of runs. This normally involves looking up a predefined OA based on the numbers of parameters, interactions and levels.
Step 4: Map the factors and values to the OA.
Step 5: Run simulations based on the OA.

Step 6: Analyze simulation results.

Nine parameters (listed in Table 1) were selected for this study. Y, P and T = Young's modulus, Poisson's ratio and thickness, respectively. PT and PF = pars tensa and pars flaccida, two regions of the TM. IMJ and ISJ = incudomallear and incudostapedial joint ligaments; SAL = stapedial annular ligament; LIG = other ligaments. The four selected interactions include Y_{PT} and T_{PT}; Y_{PT} and Y_{LIG}; Y_{PT} and Y_{LIG}; Y_{IMJ} and Y_{ISJ}

	Y_{PT}	P_{PT}	Y_{LIG}	Y_{IMJ}	Y_{ISJ}	Y_{SAL}	Y_{PF}	T_{PT}	T_{PF}
Level1	20	0.1	10	50	25	0.25	10	37	100
Level2	60	0.4	30	150	75	0.75	30	112	300

Table 1: Middle-ear parameters chosen for Taguchi analysis

The levels considered for the structures' Young's moduli and thicknesses represent an increase and decrease of the initially estimated values by 50%. Poisson's ratio levels are chosen to be 0.1and 0.4.

The OA $L_{15}\,2^{16}$ (Taguchi, 1987) is shown in Table 2. It represents 15 two-level parameters, and a total of 16 simulations.

										Output	
	Y_{PT}	T_{PT}	P_{PT}	Y_{LIG}	Y_{IMJ}	Y_{ISJ}	Y_{PF}	Y_{SAL}	T_{PF}	Vol.disp	Fp.disp
1	20	37	0.1	10	50	25	10	0.25	100	9.79	32.118
2	20	37	0.1	30	150	75	30	0.75	300	7.38	7.2817
3	60	112	0.4	10	50	25	30	0.75	300	0.976	2.0278
4	60	112	0.4	30	150	75	10	0.25	100	1.12	4.9027
5	20	112	0.4	10	150	75	10	0.75	300	2.35	5.5835
6	20	112	0.4	30	50	25	30	0.25	100	2.47	7.7572
7	60	37	0.1	10	150	75	30	0.25	100	5.52	23.066
8	60	37	0.1	30	50	25	10	0.75	300	4.63	6.2202
9	60	37	0.4	10	50	75	10	0.25	300	4.97	15.691
10	60	37	0.4	30	150	25	30	0.75	100	4.95	8.6411
11	20	112	0.1	10	50	75	30	0.75	100	2.78	7.6859
12	20	112	0.1	30	150	25	10	0.25	300	2.41	6.8268
13	60	112	0.1	10	150	25	10	0.75	100	1.27	4.3069
14	60	112	0.1	30	50	75	30	0.25	300	1.04	3.0495
15	20	37	0.4	10	150	25	30	0.25	300	7.9	18.122
16	20	37	0.4	30	50	75	10	0.75	100	9.02	11.969

Table 2: OA table for investigation and simulation results

RESULTS AND DISCUSSION

Outputs

The two outputs investigated are TM volume displacement and footplate displacement. A total of 16 static finite-element simulations were performed with SAP IV. Their results are summarized in Table 2.

Response Graphs

In the main-effects figures, nearly-horizontal lines indicate little effect. In the interaction figures, parallel lines imply no interaction.

1. Parameter effects on TM volume displacement

- Compared with the other 7 parameters, the PT thickness and Young's modulus have the greatest main effects on volume displacement (Figure 1).
- There is a very small interaction between the IMJ and ISJ (Figure 6). The interaction between PT thickness and Young's modulus is slightly larger (Figure 3). Almost parallel lines in Figures 4 and 5 indicate that there is little interaction between the parameters.

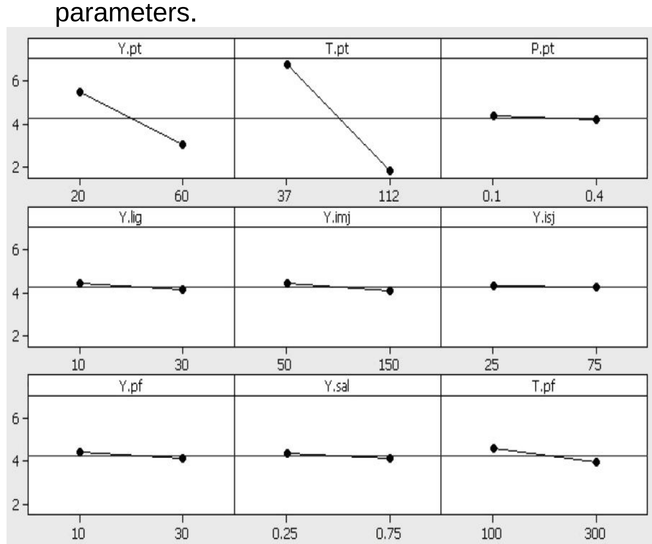

Figure 1: Parameters' main effects on TM volume displacement

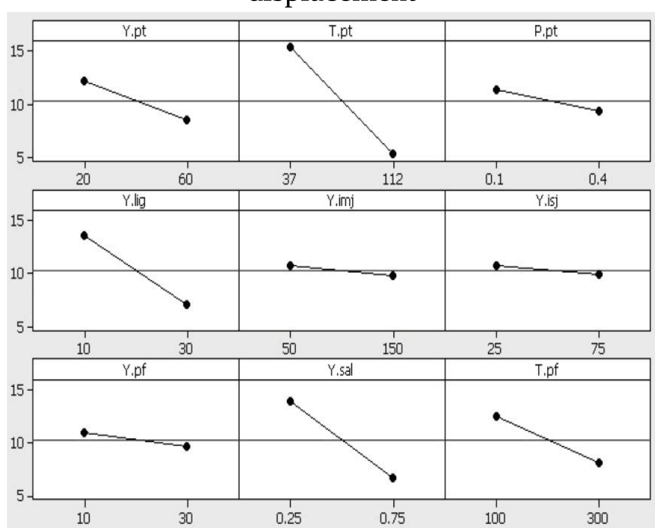

Figure 2: Parameters' main effects on footplate displacement

2. Parameter effects on footplate displacement

- Figure 2 shows that the PT thickness has the greatest effect, followed by the LIG and SAL Young's moduli, and that the rest have relatively small effects.

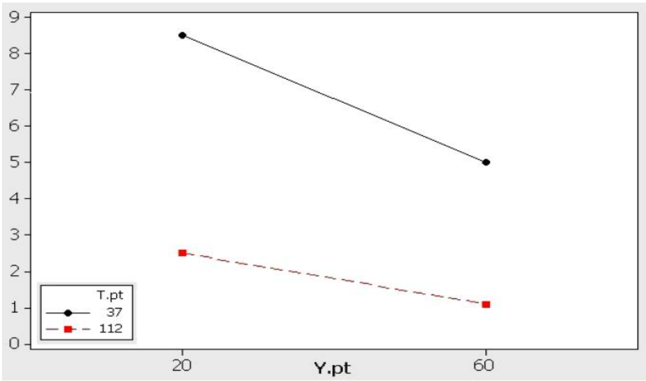

Figure 3: Interaction between Y_{PT} and T_{PT}

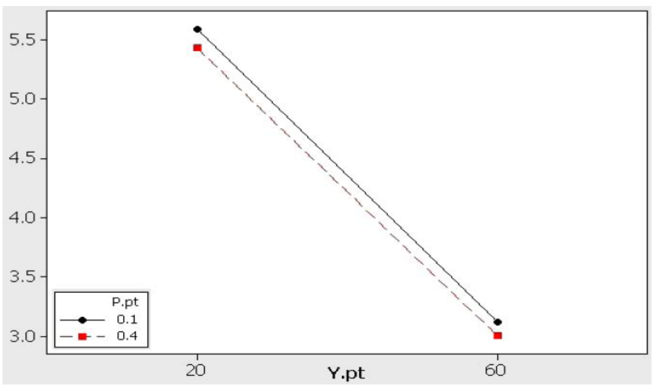

Figure 4: Interaction between Y_{PT} and P_{PT}

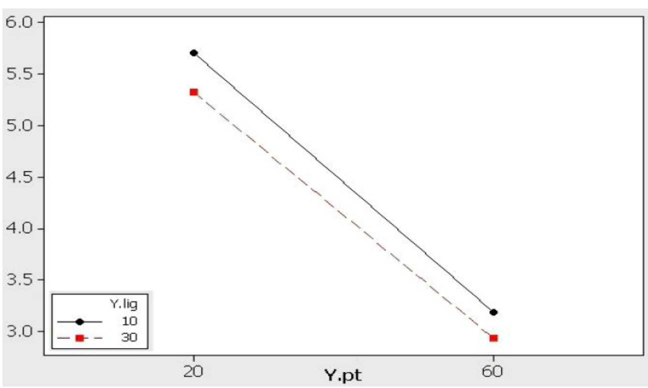

Figure 5: Interaction between Y_{PT} and Y_{LIG}

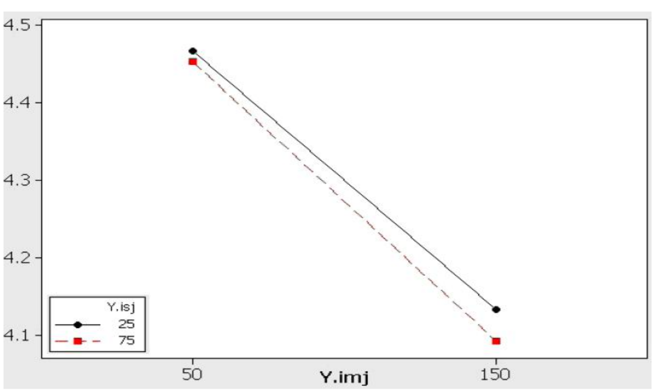

Figure 6: Interaction between Y_{IMJ} and Y_{ISJ}

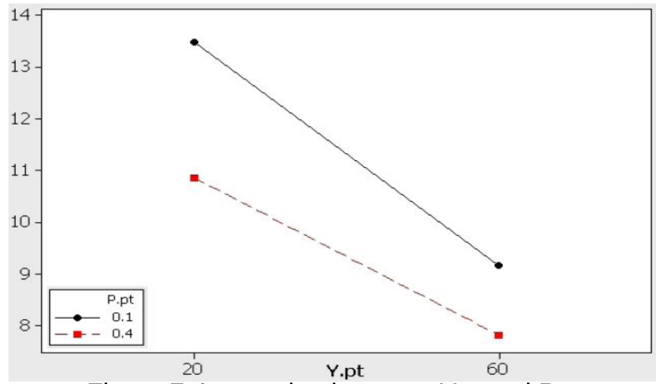

Figure 7: Interaction between Y_{PT} and P_{PT}

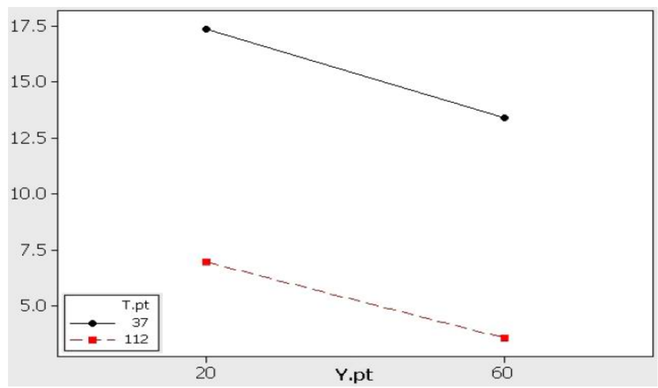

Figure 8: Interaction between Y_{PT} and T_{PT}

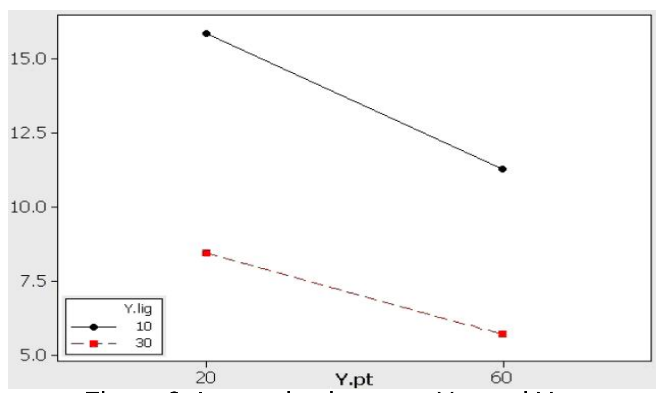

Figure 9: Interaction between Y_{PT} and Y_{LIG}

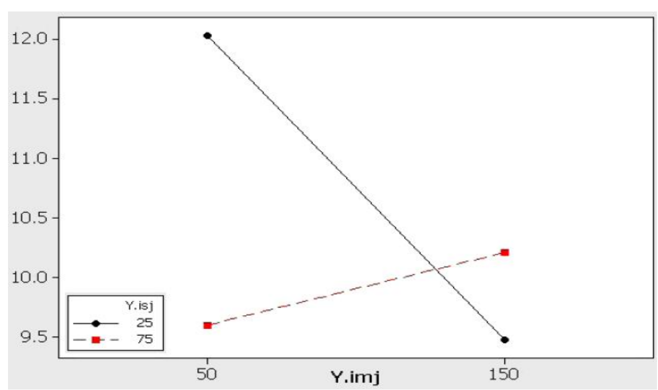

Figure 10: Interaction between Y_{IMJ} and Y_{ISJ}

- Figure 10 shows a strong interaction between the IMJ and ISJ since the lines are intersecting. These two parameters, however, have little effect on footplate displacement for the range of parameters considered here. Their effects would presumably be larger if their ranges were greater.
- Small interactions exist between PT Young's modulus and Poisson's ratio (Figure 7), and between LIG and PT Young's moduli (Figure 9).
- No interaction is seen between PT Young's modulus and thickness (Figure 8).

CONCLUSIONS

This is the first time that interactions between parameters in a middle-ear finite-element model have been studied. A strong interaction was found between the Young's moduli of the incudomallear and incudostapedial joints.

The Taguchi method is an efficient and effective method for investigating parameter sensitivity and interactions.

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