On the calculation of surface areas of objects reconstructed from serial sections

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Abstract

This paper concerns certain aspects of the calculation of the surface area of objects reconstructed from serial sections. The main points discussed are (1) the relative importance of the number of sections compared to the number of segments taken around the contour in each section; and (2) the fact that the error in the estimated surface area may actually become worse as the number of sections increases. The number of sections should be made large enough to reproduce the three-dimensional shape of the object, but not too much larger; the number of segments around the contours should then be made large enough that the segment size is similar to or smaller than the section thickness.

Introduction

A considerable amount of effort has been devoted to the problem of reconstructing the shapes of three-dimensional objects from contours traced in serial sections. In addition to simply visualizing the shapes, such reconstructions can be used for quantitative purposes such as the estimation of volumes and surface areas. In this paper I shall consider two main points arising in such work. The first point concerns the relative importance of the number of sections n compared to the number of segments k taken around the contour in each section. The second, and more important, point concerns the fact that under some conditions the error in the estimation of surface area actually becomes *worse* as the number of sections increases. I shall conclude with a discussion of some general considerations to do with surface-area estimation.

Relative importance of *n* and *k*

In a recent paper in this journal, Fahle and Palm (1983) presented a lucid review of three approaches to calculating the surface area of objects reconstructed from serial sections. In their Appendix 3, they discussed the fact that an apparently reasonable triangulation of a surface may lead to serious errors. In that discussion, one of their conclusions was that the number of sections n is less important than the number of segments k taken around the perimeter (and that this is fortunate since it is easier to make k large than it is to make n large). However, it is important to note that this conclusion is valid for their example only because it is a *cylinder*, so the contours are the same in every section – in fact, only two sections are required to represent the three-dimensional shape perfectly. This is true for any cylindrical object, regardless of the shape of the cross section, and (except near the ends) for any orientation of the planes of section (except parallel to the axis of the cylinder). In a realistic case, however, where the shape changes from slice to slice, the number of sections n will in general have an importance comparable to that of k in determining the accuracy of estimates of surface area or other quantities.

Staggering and wrinkling

The main conclusion of Appendix 3 of Fahle and Palm was that an apparently reasonable triangulation of a surface may lead to serious errors in the estimation of surface area. This is an important point that, as far as I know, had not been pointed out before. Based on their analysis they demonstrate an extraordinarily high error (almost 50%) for one set of parameters.

Examining their analysis further leads to an even more striking conclusion that they did not mention: the error in the surface-area estimate actually *increases* when the number of sections n is increased. This can be seen from their own equations, and becomes more understandable if one looks at what the triangulation they are using actually looks like.

Figure 1 shows a rendition of the front surface of a cylinder which has been divided into four layers, with each layer being triangulated differently. The bottom layer has been triangulated with $n = 2$ and $k = 11$, with the corresponding points on different sections directly in line with one another. The next layer has been triangulated with the same n and k, but the points on the different layers are staggered with respect to one another. This is the kind of triangulation pattern discussed by Fahle and Palm. It can be seen that staggering the nodes from slice to slice leads to a wrinkling of the surface, which increases the effective surface area of the object. If n is fairly small then the inclinations of the individual facets from the vertical are relatively small and the area estimate is fairly accurate. As n increases, however, the wrinkling becomes more severe (as seen in the third layer, with $n = 8$ and k still $= 11$) and the error skyrockets. The only way to bring it under control is to make k large: in the fourth layer k has been increased to 40, with n kept at 8, and it can be seen that the wrinkling has almost disappeared.

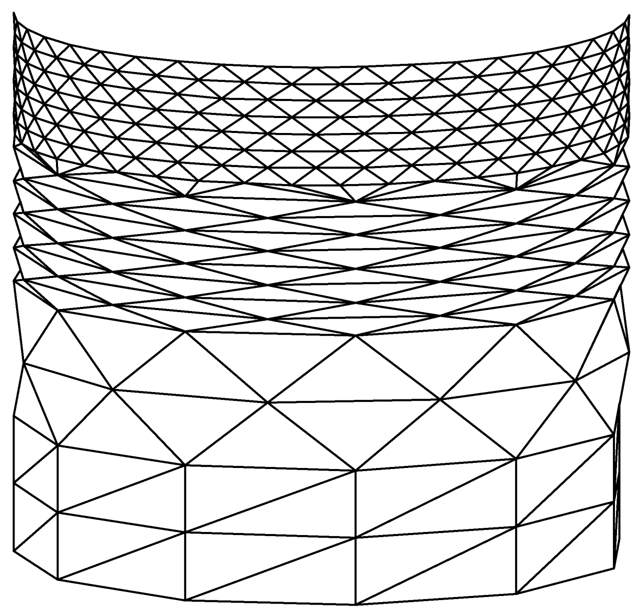

Figure 1.The front surface of a cylinder, triangulated in four layers. For the bottom two layers, $n = 2$ and $k = 11$. For the third layer n is increased to 8 while k is still 11. For the fourth layer n is kept at 8 while k is increased to 40.

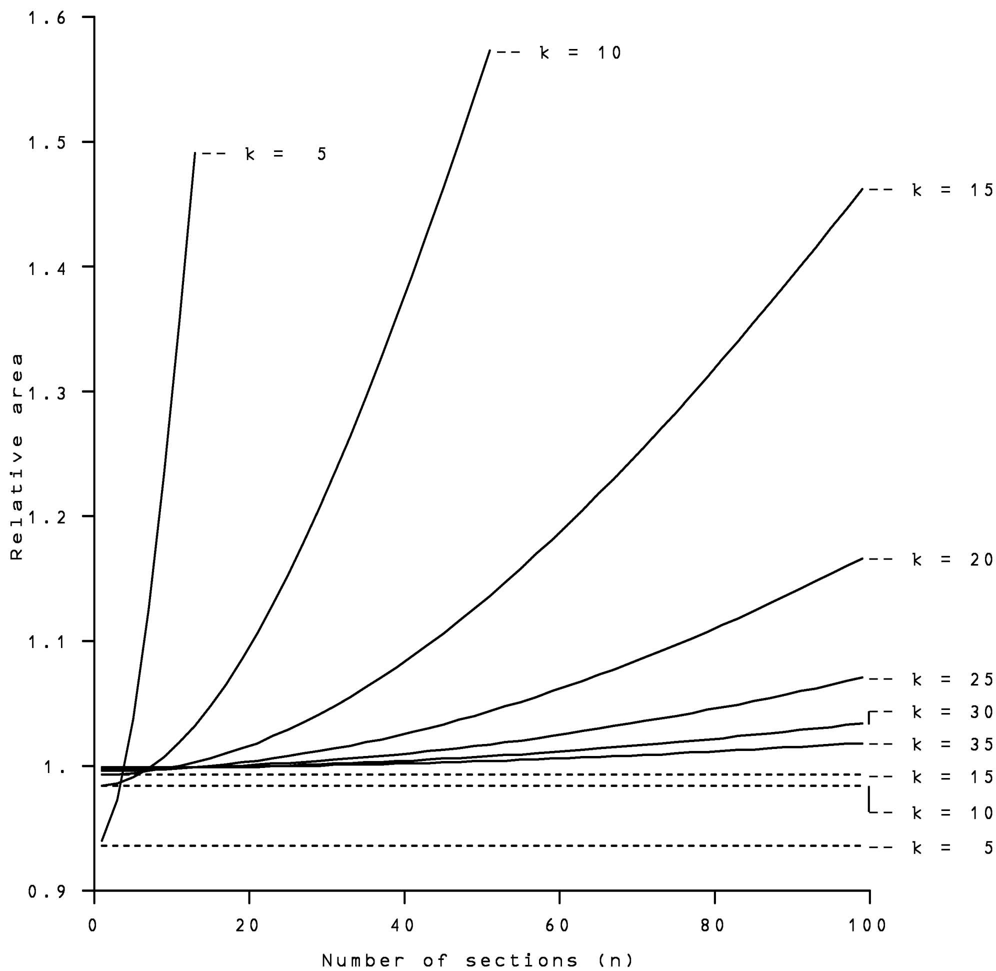

Figure 2. Relative area (estimated area divided by true area) as a function of *n*, for various values of *k*, for a triangulation with staggered nodes (solid lines) and for one with nodes that are not staggered (broken lines).

This behaviour explains the fact that, in their example with *n* and *k* both large, Fahle and Palm obtained a small error, while with *n* large and *k* small they obtained a very high error. Figure 2 shows the magnitude of the error for a cylinder as a function of *n* and *k*, both for the staggered triangulation (solid lines) and for one which is not staggered (broken lines). The formula for the surface area of a triangulated cylinder of unit length, with a circular cross section of radius *r* and with the nodes staggered from section to section, is $2\,k\,A$, where A, the area of an individual triangle, is given by $A = \frac{1}{2}[2\,r\,\sin(\pi/k)][(1/n)^2 + r^2\{1-\cos(\pi/k)\}^2]^{1/2}$. (The first term in curly brackets is the chord length for an individual triangle; the rest of the expression is the *h* of Fahle and Palm, that is, the inclined height of the triangle.) The formula for the surface area of a triangulated cylinder of unit length, with the nodes *not* staggered from section to section, is simply $2\,k\,r\,\sin(\pi/k)$, independent of n. Note that this formula always gives an underestimate of the surface area.

The main conclusion from this analysis is that *n* should be made only as large as necessary to adequately describe the variations in shape from section to section, and that *k* should then be made large enough that the widths of the triangles are comparable to (or smaller than) their heights.

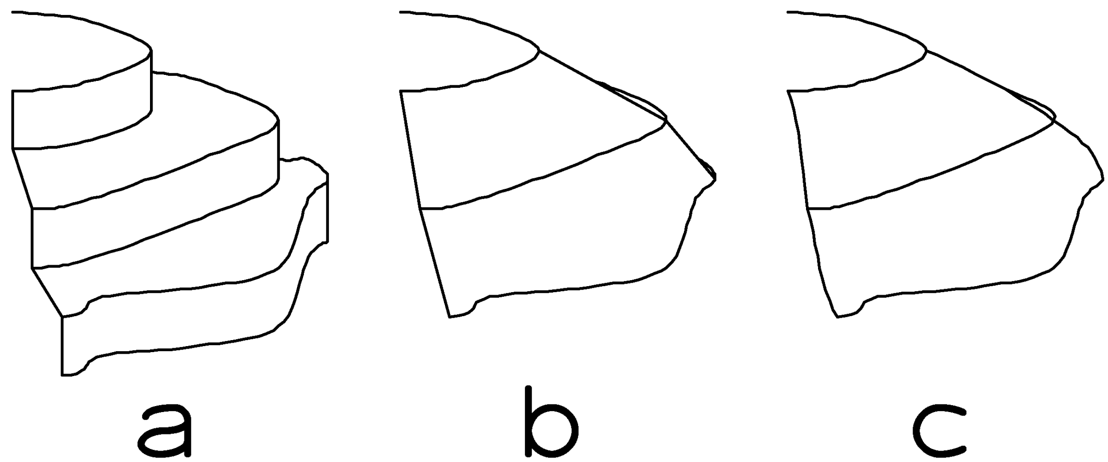

Figure 3. Schematic representation of a reconstruction with three sections, illustrating (a) a zero-order interpolation between sections, (b) a first-order (linear) interpolation, and (c) a higher-order interpolation.

General considerations

The three approaches to surface-area estimation described by Fahle and Palm represent a progression of increasingly close approximations to the actual shape of the original object. If one visualizes the plane of sectioning as being horizontal, then their first method corresponds to the common one of representing the surface of the object as a series of vertical ribbons, each following the contour of one section and having a width equal to the section thickness (Figure 3a). This can be considered as a zero-order interpolation from section to section.

Their second method is a hybrid approach leading to their third method, which represents a linear interpolation from section to section (Figure 3b). This is essentially the same as that used by Marino *et al*. (1980), except for the use of arc length rather than chord length. As Fahle and Palm pointed out, using chord length is equivalent to calculating the areas of triangles defining the surface, and if k is moderately large the difference between arc length and chord length is small. If a circle, for example, has k segments, then the arc length of a segment is $2\pi r/k$, and the chord length is $2r\sin(\pi/k)$, so the error (expressed as a fraction of the arc length) is $1-(k/\pi)\sin(\pi/k)$. For $k = 13$ the error is already less than 1%. Thus, using arc length rather than chord length has very little advantage; the fact that it does not correspond to an easily visualized surface shape may perhaps be considered to be a disadvantage.

The natural next step beyond the linear interpolation is to use some higher-order approximation (Figure 3c) such as a bicubic surface representation (Foley and van Dam, 1982, Chapter 13). This would require considerably more computation and it is not clear that the marginally improved accuracy would be worth it. One difficulty that it might help with, however, is that of extrapolating to estimate the shape of the surface before the first section and beyond the last, a question ignored by Fahle and Palm and in this paper.

If one is using the triangulation approach to process a large number of sections, it is desirable to have an automatic computerized procedure for establishing corresponding points on the different contours and for forming the required triangles. The task of recognizing corresponding points on differently shaped contours is in general a very difficult pattern-recognition problem. One approach is to arbitrarily position a fairly large number of evenly spaced points around each contour and then use some objective criterion, or ‘cost function’, for deciding which ones to join up to make triangles (Keppel, 1975; Fuchs *et al*., 1977; and others). The choice of the cost function can make a considerable difference to the shape of the resulting surface and to its surface area. In particular, a cost function which minimizes surface area will not only tend to bias the area estimate but can also lead to rather undesirable surface shapes (Funnell, 1984).

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