#815: Application of the Taguchi Method to Middle-Ear Finite-Element Modelling

Li Qi [1], Chadia S. Mikhael[1] and W. Robert J. Funnell [1, 2]

[1]Department of BioMedical Engineering [2]Department of Otolaryngology
McGill University, Montréal, QC, Canada H3A 2B4

Abstract

The quality of a finite-element model of the middle ear strongly depends on its geometry and on the choice of material properties. Uncertainty in the geometry can arise, for example, from distortion in imaging, or from the image-segmentation process due to limited spatial resolution and contrast. Likewise, uncertainty in the choice of material-property parameters can arise from lack of relevant measurements, from ear-to-ear variability, etc.

When parameter values are uncertain, the one-factor-a-time method is commonly used to investigate the effects of parameter variations; however, it does not take into account the possibility of interactions among parameters. One alternative, the full-factorial method, permits the analysis of parameter interactions but generally requires an excessive number of simulations. A more practical alternative is the Taguchi method, which was originally developed for industrial design. Via orthogonal matrices and analysis of variance (ANOVA), it determines the relative importance of each of the parameters and identifies any interactions among them.

In this work we apply the Taguchi method for the first time to a finite-element model of the middle ear, and explore its usefulness.

Introduction

The quality of a finite-element model of the middle ear strongly depends on its geometry and on the choice of material properties. Uncertainty in the geometry can arise, for example, from distortion in imaging, or from the image-segmentation process owing to limited spatial resolution and contrast. Likewise, uncertainty in the choice of material-property parameters can arise from lack of relevant measurements, from ear-to-ear variability, and so forth.

One-factor-a-time sensitivity analysis is commonly used to study the effects of parameter variations; however, it does not take into account the possibility of interactions among parameters which can affect model behaviour. Such interactions mean that the model sensitivity to one parameter can change depending on the values of other parameters. Alternatively, the full-factorial method permits the analysis of parameter interactions, but generally requires a very large number of simulations. This can be impractical when individual simulations are time-consuming. A more practical approach is the Taguchi method, which is commonly used in industry. It employs only a small number of all the possible combinations of model parameters to estimate the main effects and some interactions. An orthogonal array (OA) is used to reduce the number of simulations (Taguchi, 1987) but still obtain reasonable information. The results are quantitatively analyzed by ANOVA.

Methods

Taguchi Method

The procedure for applying the Taguchi method is as follows:

Step 1: Select parameters and interactions of interest.
Step 2: Select parameter levels.
Step 3: Find a suitable OA with the smallest number of runs. This normally involves looking up a predefined OA based on the numbers of parameters, interactions and levels.
Step 4: Map the factors and values to the OA.
Step 5: Run simulations based on the OA.
Step 6: Analyze simulation results.

A Middle-Ear Finite-Element Model

A 3-D finite-element model of an adult human middle ear was generated based on x-ray micro-Computed Tomography (CT) data with 19-µm voxels. The structures of interest were segmented using a locally developed programme, Fie. The vertices were then imported into a home-grown 3-D surface-triangulation programme, Tr3.

Both programmes are available at http://audilab.bmed.mcgill.ca/sw/.

The tympanic membrane (TM), including pars flaccida and pars tensa, was clamped at its periphery. Similarly, the ends of the mallear, incudal and stapedial ligaments were clamped. A static pressure of 1 Pa was applied normal to the TM's surface.

The model's material properties consist of the structures' Young's moduli (YM) and Poisson's ratios. Their values were obtained from the literature (Funnell, 1996; Kirikae, 1960; Koike et al., 2002; Siah, 2002) and were not adjusted to fit experimental data. Poisson's ratio has little effect on the behaviour of a middle-ear model (Funnell, 1975); all the structures are assigned the same value, 0.3.

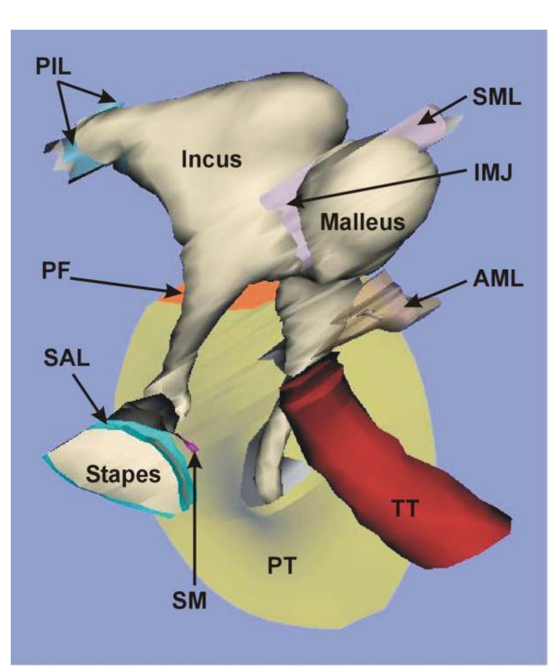

Figure 1. A 3-D model of a human middle ear.

PF: pars flaccida
PT: pars tensa
SML: superior mallear ligament
AML: anterior mallear ligament
IMJ: incudomallear joint
PIL: posterior incudal ligament
SAL: stapes annular ligament
TT: tensor tympani muscle
SM: stapedius muscle

The complete model includes 2015 nodes and 4115 triangular thin-shell elements. The finite-element simulations are done with SAP IV (Bathe et al., 1974).

The model was examined for static loading, corresponding to frequencies low enough that inertial and damping effects can be neglected. The response of this individual ear was measured using laser Doppler vibrometry at Dalhousie University. The simulated displacement for the TM is within 2% of the experimentally measured value, and the simulated displacement for the stapes is about 1/3 of the measured value. This is close enough to serve as the basis for an exploration of the effects of parameter variations.

Sensitivity Analysis

Parameter selection

Nine parameters (listed in Table 1) were selected for this study. The YM of the ossicles was not considered because of their assumed rigidity.

	Y_{PT}	P_{PT}	Y_{LIG}	Y_{IMJ}	Y_{ISJ}	Y_{SAL}	Y_{PF}	T_{PT}	T_{PF}
Level 1	20	0.1	10	50	25	0.25	10	37	100
Level 2	60	0.4	30	150	75	0.75	30	112	300

Table 1. *Middle-ear parameters chosen for Taguchi analysis. They include: pars tensa YM (Y_{PT}), its Poisson's ratio (P_{PT}), and thickness (T_{PT}); SAL YM (Y_{SAL}), IMJ YM (Y_{IMJ}), ISJ YM (Y_{ISJ}), and YM of remaining ligaments(Y_{LIG}); pars flaccida YM (Y_{PF}) and thickness (T_{PF}).*

The four selected interactions include:

- Y_{PT} and T_{PT}
- Y_{PT} and Y_{LIG}
- Y_{PF} and P_{PT}
- Y_{IMJ} and Y_{ISJ}

The levels considered for the structures' Young's moduli and thicknesses represent an increase and decrease of the initially estimated values by 50%. Poisson's ratio levels are chosen to be 0.1 and 0.4.

OA selection

The OA $L_{16}(2^{15})$ (Taguchi, 1987) is shown in Table 2. It represents 15 two-level parameters, and a total of 16 simulations.

	Y_{PT}	T_{PT}	P_{PT}	Y_{LIG}	Y_{IMJ}	Y_{ISJ}	Y_{PF}	Y_{SAL}	T_{PF}
1	20	37	0.1	10	50	25	10	0.25	100
2	20	37	0.1	30	150	75	30	0.75	300
3	60	112	0.4	10	50	25	30	0.75	300
4	60	112	0.4	30	150	75	10	0.25	100
5	20	112	0.4	10	150	75	10	0.75	300
6	20	112	0.4	30	50	25	30	0.25	100
7	60	37	0.1	10	150	75	30	0.25	100
8	60	37	0.1	30	50	25	10	0.75	300
9	60	37	0.4	10	50	75	10	0.25	300
10	60	37	0.4	30	150	25	30	0.75	100
11	20	112	0.1	10	50	75	30	0.75	100
12	20	112	0.1	30	150	25	10	0.25	300
13	60	112	0.1	10	150	25	10	0.75	100
14	60	112	0.1	30	50	75	30	0.25	300
15	20	37	0.4	10	150	25	30	0.25	300
16	20	37	0.4	30	50	75	10	0.75	100

Table 2. *OA table for investigation*

Outputs

The two outputs investigated are TM volume displacement and footplate displacement.

Results

A total of 16 static finite-element simulations were performed with SAP IV. Their results are summarized in Table 3. Graphical analysis and ANOVA were then used to analyze the simulation results.

	Output	
	TM Volume disp (pm^3)	Footplate disp (nm)
1	9.79	32.12
2	7.38	7.28
3	0.98	2.03
4	1.12	4.90
5	2.35	5.58
6	2.47	7.76
7	5.52	23.07
8	4.63	6.22
9	4.97	15.69
10	4.95	8.64
11	2.78	7.69
12	2.41	6.83
13	1.27	4.31
14	1.04	3.05
15	7.90	18.12
16	9.02	11.97

Table 3. *Simulation results*

Response Graphs

In the main-effects figures, nearly-horizontal lines indicate little effect.
In the interaction figures, parallel lines imply no interaction.

Parameter effects on TM volume displacement

- Compared with the other 7 parameters, the pars-tensa thickness and Young's modulus have the greatest main effects on volume displacement (Figure 2).
- There is a very small interaction between the IMJ and ISJ (Figure 6). The interaction between pars-tensa thickness and Young's modulus is slightly larger (Figure 3). Parallel lines in Figures 4 and 5 indicate that there is no interaction between the parameters.

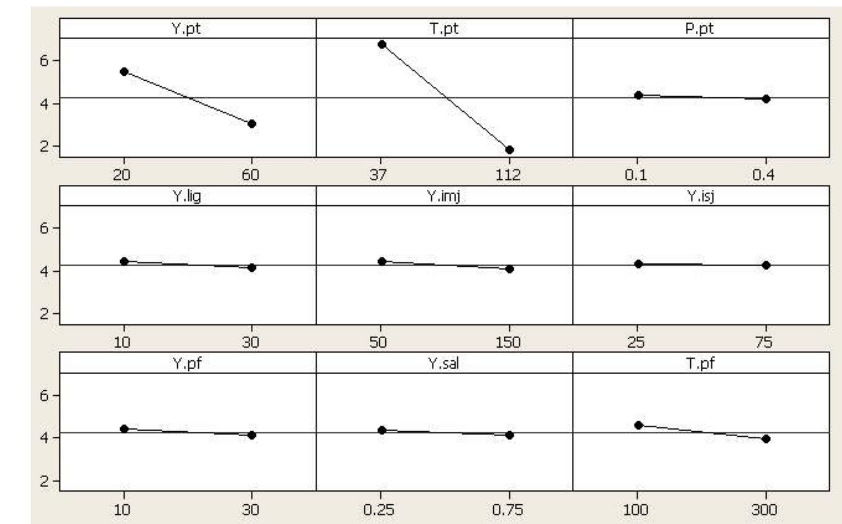

Figure 2. *Parameters' main effects on TM volume displacement*

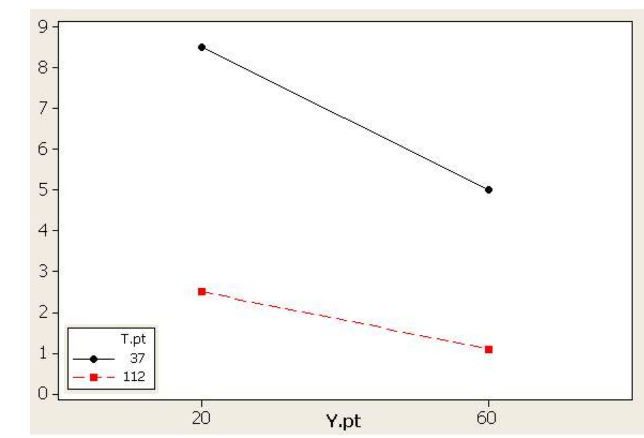

Figure 3. *Interaction between Y_{PT} and T_{PT}*

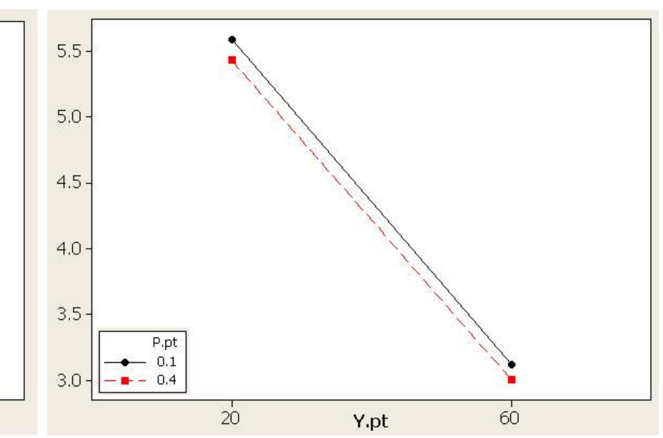

Figure 4. *Interaction between Y_{PF} and P_{PT}*

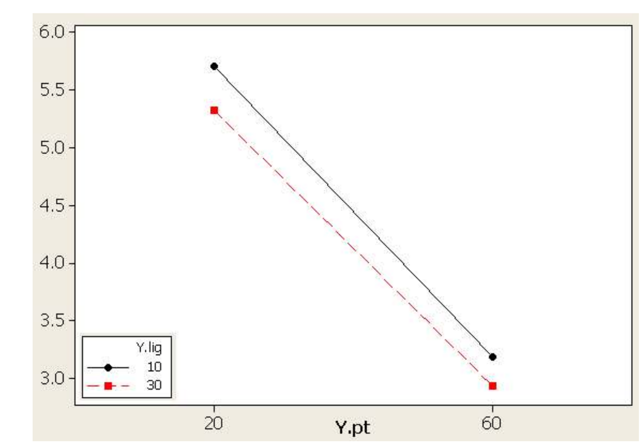

Figure 5. *Interaction between Y_{PT} and Y_{LIG}*

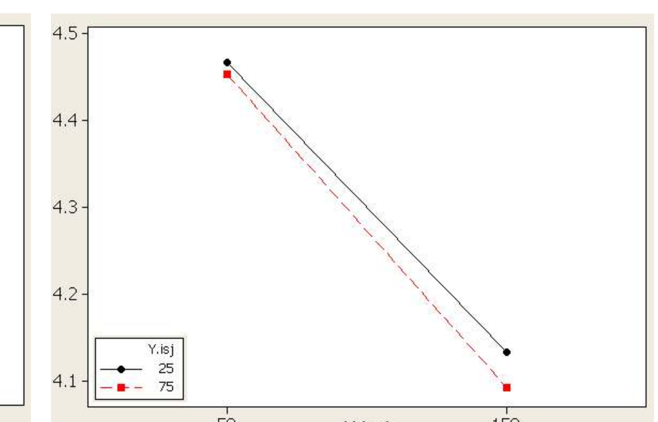

Figure 6. *Interaction between Y_{IMJ} and Y_{ISJ}*

Parameter effects on footplate displacement

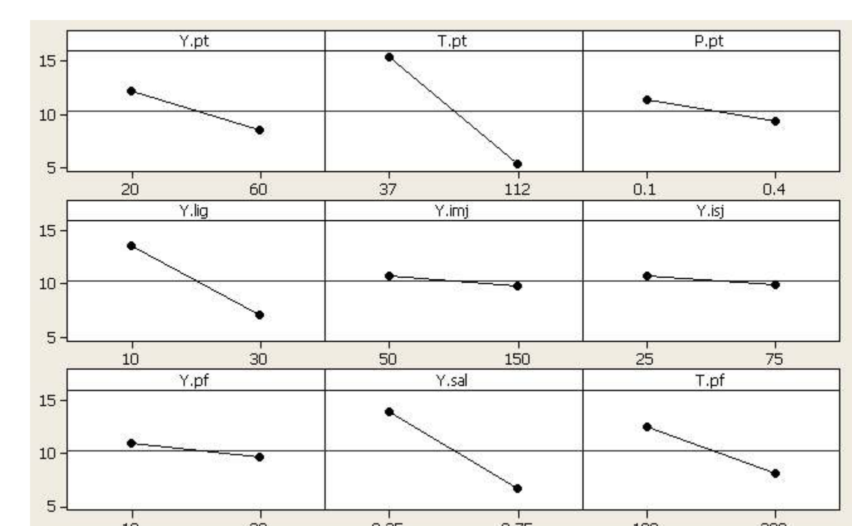

Figure 7. *Parameters' main effects on footplate displacement*

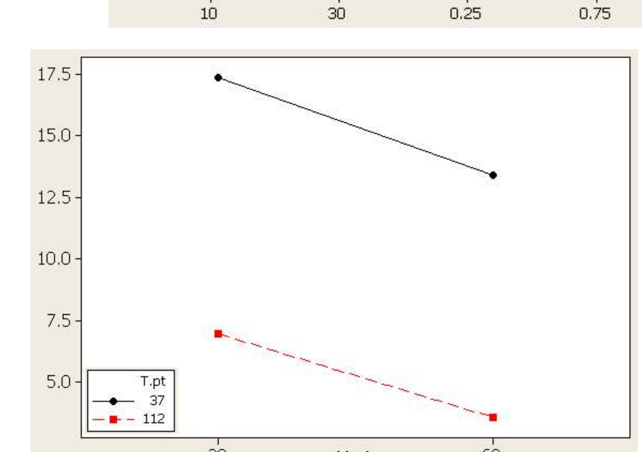

Figure 8. *Interaction between Y_{PT} and T_{PT}*

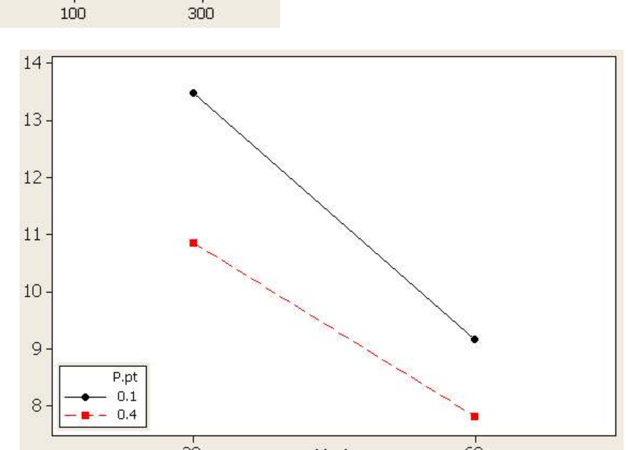

Figure 9. *Interaction between Y_{PF} and P_{PT}*

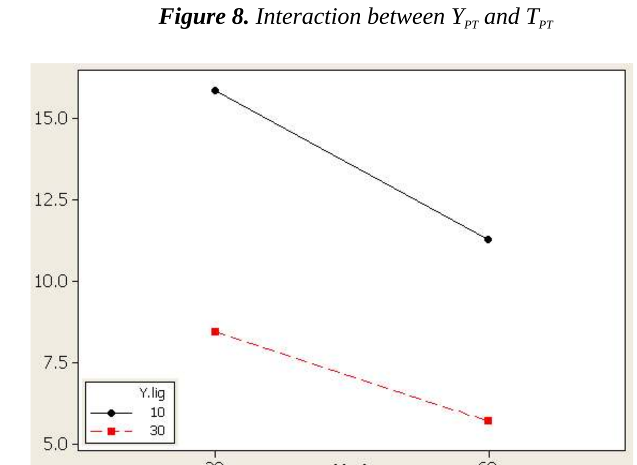

Figure 10. *Interaction between Y_{PT} and Y_{LIG}*

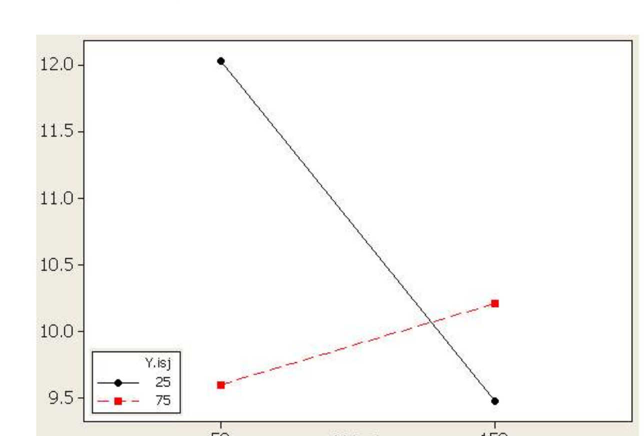

Figure 11. *Interaction between Y_{IMJ} and Y_{ISJ}*

- Figure 7 shows the effects of pars-tensa thickness and SAL YM to be the most significant. Pars-tensa thickness has the greatest effect on footplate displacement. IMJ, ISJ, and the pars-flaccida YM and Poisson's ratio have little effect. The remaining parameters have intermediate effects.
- Figure 11 shows a strong interaction between the IMJ and ISJ since the lines are intersecting. These two parameters, however, have little effect on footplate displacement for the range of parameters considered here. Their effects would presumably be larger if their ranges were greater.
- Small interactions exist between pars-tensa YM and Poisson's ratio (Figure 9), and between ligament YM and pars-tensa YM (Figure 10).
- No interaction is seen between pars-tensa YM and thickness (Figure 8).

Analysis of Variance (ANOVA)

The contribution of each parameter to the output was computed by ANOVA (Phadke, 1989). The relative effects of the parameters are summarized in Table 4. For the TM volume displacement, the pars-tensa thickness and Young's modulus represent more than 93% of the total effect.

The effects of the parameters on footplate displacement are more evenly distributed. The pars-tensa thickness has the most significant effect (>41%). The Young's moduli of the stapedius annular ligament (SAL) and the other ligaments (LIG) have similar effects, which together contribute about 37%. The rest of the parameters have less effect.

Source	Effect on TM Volume disp.	Effect on Footplate disp.
T_{PT}	75.2%	41.2%
Y_{PT}	18.3%	5.5%
P_{PT}	0.1%	1.6%
T_{PF}	0.4%	8.0%
Y_{PF}	0.3%	0.6%
Y_{LIG}	0.3%	17.0%
Y_{SAL}	0.2%	21.0%
Y_{IMJ}	0.4%	0.4%
Y_{ISJ}	0.002%	2.9%

Table 4. *ANOVA results for TM volume and footplate displacements*

Discussion

Quantitatively, the pars-tensa thickness and Young's modulus contribute more than 93% of the variation of the tympanic-membrane volume displacement, but less than 47% of that of the footplate displacement.

We observed that a strong interaction exists between the Young's moduli of the incudomallear and incudostapedial joints. It is important to take this interaction into account when the parameters' effects on model behaviour are being considered. It will be interesting to extend the analysis to include more parameter levels, wider ranges, and more interactions.

Conclusions

This is the first time that interactions between parameters in a middle-ear finite-element model have been studied.

The Taguchi method is an efficient and effective method for investigating parameter sensitivity and interactions.

Acknowledgements

We thank M. Bance (Dalhousie University, Halifax) for the middle-ear specimen and the corresponding vibration measurements.

This work was supported by the Canadian Institutes of Health Research and the Natural Sciences and Engineering Research Council (Canada).

References

1. Bathe KJ, Wilson EL, and Peterson FE (1974). *SAP IV. A structural analysis program for static and dynamic response of linear systems.* Report No. EERC 73-11, University of California, Berkeley.

2. Funnell WRJ (1996). Finite-element modeling of the cat middle ear with elastically suspended malleus and incus. 19th Midwinter Res. Mtg., Assoc. Res. Otolaryngol., St. Petersburg Beach.

3. Kirikae I (1960). *The Structure and Function of the Middle Ear.* The University of Tokyo Press.

4. Koike T, Wada H, and Kobayashi T (2002): Modeling of the human middle ear using the finite-element method. *J. Acoust. Soc. Am.*, 111, 1306-1317.

5. Siah TS (2002). *Finite Element Modeling of the Mechanics of the Coupling Between the Incus and Stapes in the Middle Ear.* M. Eng. thesis, McGill University.

6. Phadke MS (1989). *Quality Engineering using Robust Design.* Prentice Hall, New Jersey.

7. Taguchi G (1987). *System of Experimental Design.* UNIPUB Kraus International Publications, New York.

Poster available at http://audilab.bmed.mcgill.ca